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NUMERICAL SOLUTION OF FREDHOLM INTEGRAL EQUATIONS OF THE FIRST KIND WITH REAL OR COMPLEX KERNEL USING TRIANGULAR FUNCTIONS

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Abstract. Many problems in Electromagnetics are described by Fredholm integral equations of the first kind with logarithmically singular or complex kernels. An effective numerical method for solving these equations is presented. This method is based on moments method and applies triangular functions.

1. Introduction

Solving many electromagnetic problems leads to solve the Fredholm integral equations of the first kind with logarithmically singular or complex kernels[2]. These equations are inherently ill-posed problems, meaning that the solution is generally unstable[4]. Also, It can be shown that if the kernel is very smooth then the integral equation is more ill-posed.

It is the purpose of this paper to use the triangular functions (TFs) and to apply them to the method of moments. Using this method, first kind Fredholm integral equation reduces to a well-condition linear system of algebraic equations which can be solved by direct or iterative methods.

Finally, some examples of engineering interest are presented for implementing this approach.

2. Triangular functions

Triangular functions have been introduced by A. Deb et al.[3] and applied for solving a variational problem by E. Babolian and M. Salmani[1].

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Two m-sets of triangular functions (TFs) are defined over the interval [0,T) as:

$$\begin{aligned}
T1_i(t) &= \begin{cases} 1 - \frac{t-ih}{h} & ih \le t < (i+1)h, \\ 0 & \text{otherwise} \end{cases} \\
T2_i(t) &= \begin{cases} \frac{t-ih}{h} & ih \le t < (i+1)h, \\ 0 & \text{otherwise} \end{cases}
\end{aligned} \tag{2.1}$$

where $i = 0, 1, \ldots, m-1$, with a positive integer value for m. Also, consider $h = T/m$, and $T1_i$ as the ith left-handed triangular function and $T2_i$ as the ith right-handed triangular function. Also, These functions are orthogonal.

Now, consider the first m terms of left-handed and the first m terms of right-handed triangular functions and write them concisely as m-vectors:

$$\begin{aligned}
\mathbf{T1}(t) &= [T1_0(t), T1_1(t), \ldots, T1_{m-1}(t)]^t \\
\mathbf{T2}(t) &= [T2_0(t), T2_1(t), \ldots, T2_{m-1}(t)]^t
\end{aligned} \tag{2.2}$$

The expansion of a function $f(t)$ with respect to TFs, may be compactly written as:

$$\begin{aligned}
f(t) &\simeq \sum_{i=0}^{m-1} c_i T1_i(t) + \sum_{i=0}^{m-1} d_i T2_i(t) \\
&= \mathbf{c}^T \mathbf{T1}(t) + \mathbf{d}^T \mathbf{T2}(t)
\end{aligned} \tag{2.3}$$

where c_i and d_i are constant coefficients with respect to $T1_i$ and $T2_i$ for $i = 0, 1, \ldots, m-1$, respectively.

Above coefficients can be chosen by sampling $f(t)$ such that:

$$\begin{aligned}
c_i &= f(ih), \\
d_i &= f((i+1)h), \qquad \text{for } \ i = 0, 1, \ldots, m-1
\end{aligned} \tag{2.4}$$

3. Moments methods using triangular functions

In this section, the definition of triangular functions is extended over any interval $[a, b)$. Then, these functions as the basis functions are applied in the moments method.

Consider the following Fredholm integral equation of the first kind:

$$\int_a^b k(s,t)x(t)dt = y(s) \tag{3.1}$$

where $k(s,t)$ and $y(s)$ are known functions but $x(t)$ is not.

Approximating the function $x(s)$ with respect to triangular functions by (2.3) gives:

$$x(s) \simeq \mathbf{c}^T \mathbf{T1}(s) + \mathbf{d}^T \mathbf{T2}(s) \tag{3.2}$$

Substituting Eq.(3.2) into (3.1) follows:

$$\mathbf{c}^T \int_a^b k(s,t)\mathbf{T1}(t)dt + \mathbf{d}^T \int_a^b k(s,t)\mathbf{T2}(t)dt \simeq y(s) \tag{3.3}$$

Now, let s_i, $i = 0, 1, \ldots, 2m-1$ be $2m$ appropriate points in interval $[a, b)$; putting $s = s_i$ in Eq.(3.3), follows:

$$\sum_{j=0}^{m-1} \left[c_j \int_a^b k(s_i,t)T1_j(t)dt + d_j \int_a^b k(s_i,t)T2_j(t)dt \right] \simeq y(s_i), \qquad i = 0, 1, \ldots, 2m-1 \tag{3.4}$$

Now, replace $\simeq$ with $=$, hence Eq.(3.4) is a linear system of $2m$ algebraic equations for $2m$ unknown components. So, an approximate solution $x(s) \simeq \mathbf{c}^T\mathbf{T1}(s) + \mathbf{d}^T\mathbf{T2}(s)$, is obtained for Eq.(3.1).

Note that with using (2.4), the number of unknown coefficients in (3.3) can be reduced to $m+1$, therefore it should be considered just $m+1$ equations with selecting $m+1$ appropriate points in interval $[a, b)$. Computation of the unknowns using these two methods are also compared.

4. Conclusion

The presented method is applied to solve some electromagnetic problems such as determining the charge distribution, electromagnetic scattering, and determining the radar cross section. The numerical results show efficiency and accuracy of this method. Also, this approach can be generalized to apply to objects of arbitrary geometry and arbitrary material.

References

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