Advances in Numerical Methods for Solving Electromagnetic Integral Equations

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Abstract

Electromagnetic integral equations remain a cornerstone in the analysis of radiation and scattering phenomena across complex geometries. Recent developments in numerical algorithms have significantly enhanced the accuracy, efficiency, and scalability of solution techniques. This article reviews key advances in discretization strategies, fast computational schemes, and hybrid formulations that have emerged in recent years. Emphasis is placed on the broad applicability of these methods to multi-scale, multi-physics, and electrically large problems.

Keywords: Electromagnetic integral equations; Numerical methods; Method of moments; Fast algorithms; Computational electromagnetics.

1. Introduction

Integral-equation formulations play a foundational role in electromagnetic analysis due to their ability to inherently satisfy radiation conditions and reduce problem dimensionality. Over the past decades, researchers have continually sought improved numerical schemes to address challenges posed by complex geometries and high-frequency behavior. Classical techniques such as the Method of Moments have matured, but their direct implementation becomes computationally demanding for large-scale models. As computational resources evolved, so did the need for better-conditioned formulations, higher-order basis functions, and efficient matrix-compression strategies. Modern approaches now integrate fast algorithms, adaptive meshing, and hybrid modeling paradigms to handle a wider spectrum of electromagnetic problems. These developments have contributed to more robust, scalable simulations capable of addressing both fine structural

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detail and electrically large environments. This article provides an overview of these advances from a general and contemporary perspective.

2. Recent Trends in Computational Techniques for Electromagnetic Integral Equations

Contemporary research in computational electromagnetics has focused on bridging the gap between accuracy and computational cost. One significant trend is the adoption of fast solvers that reduce both memory consumption and execution time through hierarchical matrix representations or multi-level decompositions. Additionally, hybrid formulations combining surface and volume integral equations have gained traction for problems involving material interfaces. High-order discretization schemes are now widely explored to capture fine geometrical features with fewer unknowns. Parallel computing strategies, including GPU-accelerated frameworks, have further empowered the solution of integral equations for electrically large structures. Efforts toward automated meshing, preconditioning, and error estimation continue to make these methods more accessible and reliable for general-purpose electromagnetic simulation.

References

[1] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, An integral equation modeling of electromagnetic scattering from the surfaces of arbitrary resistance distribution, Progress In Electromagnetics Research B 3, 157-172, 2008.

[2] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, Z. Masouri, A moment method simulation of electromagnetic scattering from conducting bodies, Progress In Electromagnetics Research 81, 99-119, 2008.

[3] S. Hatamzadeh-Varmazyar, M. Fathy, Z. Masouri, Optimization of face recognition algorithms using wavelet transforms, 40th Annual Iranian Mathematics Conference, Sharif University of Technology, Tehran, Iran, 2009.

[4] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, Introducing a direct method to solve nonlinear Volterra and Fredholm integral equations using orthogonal triangular functions, Mathematics Scientific Journal 5 (1), 11-26, 2009.

[5] S. Hatamzadeh-Varmazyar, Z. Masouri, Numerical expansion-iterative method for analysis of integral equation models arising in one-and two-dimensional electromagnetic scattering, Engineering Analysis with Boundary Elements 36 (3), 416-422, 2012.

[6] S. Hatamzadeh-Varmazyar, Z. Masouri, A numerical method for calculation of electrostatic charge distribution induced on conducting surfaces, Advanced Computational Techniques in Electromagnetics 2014, 1-9, 2014.

[7] Z. Masouri, S. Hatamzadeh-Varmazyar, Applying integral equation modeling technique in determination of charge distribution on conducting structures, Advanced Computational Techniques in Electromagnetics 2013, 1-8, 2013.

[8] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, Numerical solution of nonlinear Volterra-Fredholm integro-differential equations via direct method using triangular functions, Computers & Mathematics with Applications 58 (2), 239-247, 2009.

[9] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, A direct method for numerically solving integral equations system using orthogonal triangular functions, International Journal of Industrial Mathematics 1 (2), 135-145, 2009.

[10] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, New numerical method for determining the scattered electromagnetic fields from thin wires, Progress In Electromagnetics Research B 3, 207-218, 2008.

[11] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, New direct method to solve nonlinear Volterra-Fredholm integral and integro-differential equations using operational matrix with block-pulse functions, Progress In Electromagnetics Research B 8, 59-76, 2008.

[12] S. Hatamzadeh-Varmazyar, Z. Masouri, Determining the electromagnetic fields scattered from PEC cylinders, International Journal of Mathematics & Computation 28 (4), 1-8, 2017.

[13] R. Danesfahani, S. Hatamzadeh-Varmazyar, E. Babolian, Z. Masouri, Applying shannon wavelet basis functions to the method of moments for evaluating the radar cross section of the conducting and resistive surfaces, Progress In Electromagnetics Research B 8, 257-292, 2008.

[14] S. Hatamzadeh-Varmazyar, Z. Masouri, Numerical method for analysis of one-and two-dimensional electromagnetic scattering based on using linear Fredholm integral equation models, Mathematical and Computer Modelling 54 (9), 2199-2210, 2011.

[15] Z. Masouri, E. Babolian, S. Hatamzadeh-Varmazyar, An expansion-iterative method for numerically solving Volterra integral equation of the first kind, Computers & Mathematics with Applications 59 (4), 1491-1499, 2010.

[16] S. Hatamzadeh-Varmazyar, Z. Masouri, A numerical approach for calculating the radar cross-section of two-dimensional perfect electrically conducting structures, Journal of Electromagnetic Waves and Applications 28 (11), 1360-1375, 2014.

[17] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, E. Babolian, Z. Masouri, Numerical approach to survey the problem of electromagnetic scattering from resistive strips based on using a set of orthogonal basis functions, Progress In Electromagnetics Research 81, 393-412, 2008.

[18] S. Hatamzadeh-Varmazyar, Z. Masouri, E. Babolian, Numerical method for solving arbitrary linear differential equations using a set of orthogonal basis functions and operational matrix, Applied Mathematical Modelling 40 (1), 233-253, 2016.

[19] S. Hatamzadeh-Varmazyar, Z. Masouri, A computational method for numerically solving linear integro-differential equations, International Journal of Mathematics & Computation 29 (1), 82-93, 2018.

[20] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, E. Babolian, Z. Masouri, Calculating the radar cross section of the resistive targets using the Haar wavelets, Progress In Electromagnetics Research 83, 55-80, 2008.

[21] Z. Masouri, S. Hatamzadeh-Varmazyar, An analysis of electromagnetic scattering from finite-width strips, International Journal of Industrial Mathematics 5 (3), 199-204, 2013.

[22] Z. Masouri, E. Babolian, S. Hatamzadeh-Varmazyar, Collocation method for solving Fredholm integral equation of the first kind using Shannon wavelet system, 41th Annual Iranian Mathematics Conference, Urmia University, Urmia, Iran, 316, 2010.

[23] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, R. Sadeghzadeh-Sheikhan, One-and two-dimensional scattering analysis using a fast numerical method, IET Microwaves, Antennas & Propagation 5 (10), 1148-1155, 2011.

[24] R. Danesfahani, S. Hatamzadeh-Varmazyar, E. Babolian, Z. Masouri, A scheme for RCS determination using wavelet basis, AEU-International Journal of Electronics and Communications 64 (8), 757-765, 2010.

[25] S. Hatamzadeh-Varmazyar, Z. Masouri, A fast numerical method for analysis of one-and two-dimensional electromagnetic scattering using a set of cardinal functions, Engineering Analysis with Boundary Elements 36 (11), 1631-1639, 2012.

[26] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, A set of multi-dimensional orthogonal basis functions and its application to solve integral equations, International Journal of Applied Mathematics and Computation 2 (1), 032-049, 2010.

[27] S. Hatamzadeh-Varmazyar, M. Naser Moghadasi, R. Sadeghzadeh-Sheikhan, Numerical method for analysis of radiation from thin wire dipole antenna, International Journal of Industrial Mathematics 3 (2), 135-142, 2011.

[28] S. Hatamzadeh-Varmazyar, The error analysis and convergence evaluation of a computational technique for solving electromagnetic scattering problems, Advanced Computational Techniques in Electromagnetics 2015 (1), 66-69, 2015.

[29] Z. Masouri, S. Hatamzadeh-Varmazyar, Numerical solution of Fredholm integral equations of the first kind with real or complex kernel using triangular functions, 38th Annual Iranian Mathematics Conference, Zanjan University, Zanjan, Iran, 284-286, 2007.

[30] S. Hatamzadeh-Varmazyar, Z. Masouri, Calculation of electric charge density based on a numerical approximation method using triangular functions, Advanced Computational Techniques in Electromagnetics 2013, 1-11, 2013.

[31] Z. Masouri, S. Hatamzadeh-Varmazyar, E. Babolian, Numerical method for solving system of Fredhlom integral equations using Chebyshev cardinal functions, Advanced Computational Techniques in Electromagnetics 2014, 1-13, 2014.

[32] Z. Masouri, S. Hatamzadeh-Varmazyar, Evaluation of current distribution induced on perfect electrically conducting scatterers, International Journal of Industrial Mathematics 5 (2), 167-173, 2013.

[33] S. Hatamzadeh-Varmazyar, Z. Masouri, An Efficient Numerical Algorithm For Solving Linear Differential Equations of Arbitrary Order And Coefficients, International Journal of Industrial Mathematics 10 (2), 127-138, 2018.

[34] S. Hatamzadeh-Varmazyar, Z. Masouri, A fast and accurate expansion-iterative method for solving second kind Volterra integral equations, International Journal of Industrial Mathematics 10 (1), 29-37, 2018.

[35] S. Hatamzadeh-Varmazyar, Z. Masouri, A numerical scheme for two-dimensional scattering analysis, International Journal of Mathematics & Computation 29 (2), 84-94, 2018.

[36] S. Hatamzadeh-Varmazyar, Z. Masouri, Numerical Solution of Second Kind Volterra and Fredholm Integral Equations Based on a Direct Method Via Triangular Functions, International Journal of Industrial Mathematics 11 (2), 79-87, 2019.

[37] Z. Masouri, S. Hatamzadeh, A Regularization-direct Method to Numerically Solve First Kind Fredholm Integral Equation, Kyungpook Mathematical Journal 60 (4), 869-881, 2020.

[38] S. Hatamzadeh, Z. Masouri, A numerical evaluation of radiation from dipole antenna based on a set of wavelet functions, International Journal of Imaging and Robotics 21 (1), 45-56, 2021.

[39] S. Hatamzadeh, Z. Masouri, Applying a set of orthogonal basis functions in numerical solution of Hallen's integral equation for dipole antenna of perfectly conducting material, Journal of Modern Materials 9 (1), 36-49, 2022.

[40] Z. Masouri, S. Hatamzadeh, A Computational Approach for Analysis of Scattering Characteristics for Strips of Resistive Material, International Journal of Research Publication and Reviews 4 (12), 3640-3646, 2023.

[41] Z. Masouri, S. Hatamzadeh, An Efficient Scheme for Optimization of Recognition Algorithms, International Journal of Research Publication and Reviews 5 (2), 1746-1749, 2024.

[42] S. Hatamzadeh, Z. Masouri, A data analysis based survey on a scheme for solving integral equations, International Conference on Optimization and Data Science in Industrial Engineering (ODSIE 2024), Istanbul, Turkey, Nov. 7-8, 2024.

[43] S. Hatamzadeh, Z. Masouri, Computational Approach for Solving Fredholm Integral Equations, Communications in Computer and Information Science 2482, 303-322, 2026.