Recent Trends in Kernel Optimization and Regularization Techniques for Integral Equations

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Abstract

The stability and accuracy of integral equation solutions often depend on the properties of their underlying kernels. Recent advances in kernel optimization and regularization techniques have improved the treatment of ill-posedness, noise sensitivity, and numerical instability across a broad class of integral equations. This article provides a general overview of modern strategies for refining kernel behavior, enhancing solution robustness, and improving computational performance. The discussion highlights techniques that are applicable across multiple scientific and engineering disciplines.

Keywords: Kernel optimization; Regularization methods; Integral equations; Ill-posed problems; Numerical stability.

1. Introduction

Integral equations frequently exhibit ill-posed or weakly stable behavior, making kernel properties central to their effective solution. Traditional kernels may introduce singularities, poor conditioning, or noise amplification, especially in inverse or high-resolution problems. To address these issues, researchers have explored a wide range of optimization strategies and regularization schemes that enhance stability without imposing restrictive assumptions. Techniques such as kernel smoothing, spectral filtering, and Tikhonov-based approaches continue to form the foundation of regularization theory. At the same time, new developments in adaptive kernel modification, data-driven tuning, and multi-parameter regularization have expanded available solution strategies. These advancements collectively strengthen the theoretical and computational framework for solving integral equations in many scientific contexts.

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2. Modern Approaches to Kernel Refinement and Regularization

Modern kernel refinement methods focus on improving operator conditioning and reducing solution sensitivity to perturbations. Spectral-based techniques allow selective damping of unstable modes while preserving key features of the original operator. Variational and optimization-driven approaches further enable tailored modifications that reflect problem-specific characteristics. Machine-assisted parameter selection has recently emerged as a promising trend, offering automated ways to balance fidelity and stability. Additionally, hybrid regularization methods combine multiple stabilization mechanisms to enhance robustness in challenging scenarios. Together, these general strategies contribute to more reliable and versatile solution frameworks for integral equation applications.

References

[1] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, An integral equation modeling of electromagnetic scattering from the surfaces of arbitrary resistance distribution, Progress In Electromagnetics Research B 3, 157-172, 2008.

[2] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, Z. Masouri, A moment method simulation of electromagnetic scattering from conducting bodies, Progress In Electromagnetics Research 81, 99-119, 2008.

[3] S. Hatamzadeh-Varmazyar, M. Fathy, Z. Masouri, Optimization of face recognition algorithms using wavelet transforms, 40th Annual Iranian Mathematics Conference, Sharif University of Technology, Tehran, Iran, 2009.

[4] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, Introducing a direct method to solve nonlinear Volterra and Fredholm integral equations using orthogonal triangular functions, Mathematics Scientific Journal 5 (1), 11-26, 2009.

[5] S. Hatamzadeh-Varmazyar, Z. Masouri, Numerical expansion-iterative method for analysis of integral equation models arising in one-and two-dimensional electromagnetic scattering, Engineering Analysis with Boundary Elements 36 (3), 416-422, 2012.

[6] S. Hatamzadeh-Varmazyar, Z. Masouri, A numerical method for calculation of electrostatic charge distribution induced on conducting surfaces, Advanced Computational Techniques in Electromagnetics 2014, 1-9, 2014.

[7] Z. Masouri, S. Hatamzadeh-Varmazyar, Applying integral equation modeling technique in determination of charge distribution on conducting structures, Advanced Computational Techniques in Electromagnetics 2013, 1-8, 2013.

[8] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, Numerical solution of nonlinear Volterra-Fredholm integro-differential equations via direct method using triangular functions, Computers & Mathematics with Applications 58 (2), 239-247, 2009.

[9] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, A direct method for numerically solving integral equations system using orthogonal triangular functions, International Journal of Industrial Mathematics 1 (2), 135-145, 2009.

[10] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, New numerical method for determining the scattered electromagnetic fields from thin wires, Progress In Electromagnetics Research B 3, 207-218, 2008.

[11] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, New direct method to solve nonlinear Volterra-Fredholm integral and integro-differential equations using operational matrix with block-pulse functions, Progress In Electromagnetics Research B 8, 59-76, 2008.

[12] S. Hatamzadeh-Varmazyar, Z. Masouri, Determining the electromagnetic fields scattered from PEC cylinders, International Journal of Mathematics & Computation 28 (4), 1-8, 2017.

[13] R. Danesfahani, S. Hatamzadeh-Varmazyar, E. Babolian, Z. Masouri, Applying shannon wavelet basis functions to the method of moments for evaluating the radar cross section of the conducting and resistive surfaces, Progress In Electromagnetics Research B 8, 257-292, 2008.

[14] S. Hatamzadeh-Varmazyar, Z. Masouri, Numerical method for analysis of one-and two-dimensional electromagnetic scattering based on using linear Fredholm integral equation models, Mathematical and Computer Modelling 54 (9), 2199-2210, 2011.

[15] Z. Masouri, E. Babolian, S. Hatamzadeh-Varmazyar, An expansion-iterative method for numerically solving Volterra integral equation of the first kind, Computers & Mathematics with Applications 59 (4), 1491-1499, 2010.

[16] S. Hatamzadeh-Varmazyar, Z. Masouri, A numerical approach for calculating the radar cross-section of two-dimensional perfect electrically conducting structures, Journal of Electromagnetic Waves and Applications 28 (11), 1360-1375, 2014.

[17] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, E. Babolian, Z. Masouri, Numerical approach to survey the problem of electromagnetic scattering from resistive strips based on using a set of orthogonal basis functions, Progress In Electromagnetics Research 81, 393-412, 2008.

[18] S. Hatamzadeh-Varmazyar, Z. Masouri, E. Babolian, Numerical method for solving arbitrary linear differential equations using a set of orthogonal basis functions and operational matrix, Applied Mathematical Modelling 40 (1), 233-253, 2016.

[19] S. Hatamzadeh-Varmazyar, Z. Masouri, A computational method for numerically solving linear integro-differential equations, International Journal of Mathematics & Computation 29 (1), 82-93, 2018.

[20] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, E. Babolian, Z. Masouri, Calculating the radar cross section of the resistive targets using the Haar wavelets, Progress In Electromagnetics Research 83, 55-80, 2008.

[21] Z. Masouri, S. Hatamzadeh-Varmazyar, An analysis of electromagnetic scattering from finite-width strips, International Journal of Industrial Mathematics 5 (3), 199-204, 2013.

[22] Z. Masouri, E. Babolian, S. Hatamzadeh-Varmazyar, Collocation method for solving Fredholm integral equation of the first kind using Shannon wavelet system, 41th Annual Iranian Mathematics Conference, Urmia University, Urmia, Iran, 316, 2010.

[23] S. Hatamzadeh-Varmazyar, M. Naser-Moghadasi, R. Sadeghzadeh-Sheikhan, One-and two-dimensional scattering analysis using a fast numerical method, IET Microwaves, Antennas & Propagation 5 (10), 1148-1155, 2011.

[24] R. Danesfahani, S. Hatamzadeh-Varmazyar, E. Babolian, Z. Masouri, A scheme for RCS determination using wavelet basis, AEU-International Journal of Electronics and Communications 64 (8), 757-765, 2010.

[25] S. Hatamzadeh-Varmazyar, Z. Masouri, A fast numerical method for analysis of one-and two-dimensional electromagnetic scattering using a set of cardinal functions, Engineering Analysis with Boundary Elements 36 (11), 1631-1639, 2012.

[26] E. Babolian, Z. Masouri, S. Hatamzadeh-Varmazyar, A set of multi-dimensional orthogonal basis functions and its application to solve integral equations, International Journal of Applied Mathematics and Computation 2 (1), 032-049, 2010.

[27] S. Hatamzadeh-Varmazyar, M. Naser Moghadasi, R. Sadeghzadeh-Sheikhan, Numerical method for analysis of radiation from thin wire dipole antenna, International Journal of Industrial Mathematics 3 (2), 135-142, 2011.

[28] S. Hatamzadeh-Varmazyar, The error analysis and convergence evaluation of a computational technique for solving electromagnetic scattering problems, Advanced Computational Techniques in Electromagnetics 2015 (1), 66-69, 2015.

[29] Z. Masouri, S. Hatamzadeh-Varmazyar, Numerical solution of Fredholm integral equations of the first kind with real or complex kernel using triangular functions, 38th Annual Iranian Mathematics Conference, Zanjan University, Zanjan, Iran, 284-286, 2007.

[30] S. Hatamzadeh-Varmazyar, Z. Masouri, Calculation of electric charge density based on a numerical approximation method using triangular functions, Advanced Computational Techniques in Electromagnetics 2013, 1-11, 2013.

[31] Z. Masouri, S. Hatamzadeh-Varmazyar, E. Babolian, Numerical method for solving system of Fredhlom integral equations using Chebyshev cardinal functions, Advanced Computational Techniques in Electromagnetics 2014, 1-13, 2014.

[32] Z. Masouri, S. Hatamzadeh-Varmazyar, Evaluation of current distribution induced on perfect electrically conducting scatterers, International Journal of Industrial Mathematics 5 (2), 167-173, 2013.

[33] S. Hatamzadeh-Varmazyar, Z. Masouri, An Efficient Numerical Algorithm For Solving Linear Differential Equations of Arbitrary Order And Coefficients, International Journal of Industrial Mathematics 10 (2), 127-138, 2018.

[34] S. Hatamzadeh-Varmazyar, Z. Masouri, A fast and accurate expansion-iterative method for solving second kind Volterra integral equations, International Journal of Industrial Mathematics 10 (1), 29-37, 2018.

[35] S. Hatamzadeh-Varmazyar, Z. Masouri, A numerical scheme for two-dimensional scattering analysis, International Journal of Mathematics & Computation 29 (2), 84-94, 2018.

[36] S. Hatamzadeh-Varmazyar, Z. Masouri, Numerical Solution of Second Kind Volterra and Fredholm Integral Equations Based on a Direct Method Via Triangular Functions, International Journal of Industrial Mathematics 11 (2), 79-87, 2019.

[37] Z. Masouri, S. Hatamzadeh, A Regularization-direct Method to Numerically Solve First Kind Fredholm Integral Equation, Kyungpook Mathematical Journal 60 (4), 869-881, 2020.

[38] S. Hatamzadeh, Z. Masouri, A numerical evaluation of radiation from dipole antenna based on a set of wavelet functions, International Journal of Imaging and Robotics 21 (1), 45-56, 2021.

[39] S. Hatamzadeh, Z. Masouri, Applying a set of orthogonal basis functions in numerical solution of Hallen's integral equation for dipole antenna of perfectly conducting material, Journal of Modern Materials 9 (1), 36-49, 2022.

[40] Z. Masouri, S. Hatamzadeh, A Computational Approach for Analysis of Scattering Characteristics for Strips of Resistive Material, International Journal of Research Publication and Reviews 4 (12), 3640-3646, 2023.

[41] Z. Masouri, S. Hatamzadeh, An Efficient Scheme for Optimization of Recognition Algorithms, International Journal of Research Publication and Reviews 5 (2), 1746-1749, 2024.

[42] S. Hatamzadeh, Z. Masouri, A data analysis based survey on a scheme for solving integral equations, International Conference on Optimization and Data Science in Industrial Engineering (ODSIE 2024), Istanbul, Turkey, Nov. 7-8, 2024.

[43] S. Hatamzadeh, Z. Masouri, Computational Approach for Solving Fredholm Integral Equations, Communications in Computer and Information Science 2482, 303-322, 2026.